

# **ENGN1750 *Advanced Mechanics of Solids***

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Course Website

<http://www.brown.edu/Departments/Engineering/Labs/Gudurulab/ENGN1750/>

Tuesday, Thursday; 2:30 – 3:50pm, B&H 159

- Weekly section will be held in the instructional computer lab.

# Course Outline

## **1. Introduction**

- 1.1 Scope of the course
- 1.2 Basic concepts of solid mechanics
- 1.3 Overview of finite element method in computational solid mechanics

## **2. Mathematical background**

- 2.1 Vector algebra
- 2.2 Index notations
- 2.3 Matrices and tensors
- 2.4 Vector and tensor calculus

## **3. Stress in a solid**

- 3.1 Body forces, surface forces and traction vector at a point on the surface
- 3.2 Stress tensor at a point
- 3.3 Principal stresses at a point
- 3.4 Balance of momentum and equilibrium equations

## **4. Strain in a solid**

- 4.1 Displacement field in a deformed solid
- 4.2 Strain tensor in a Solid
- 4.3 Principal strains at a point
- 4.4 Compatibility conditions on a strain field
- 4.5 Principal strains at a point

## **5. Mechanical Behavior of Solids**

- 5.1 Role of experiments in solid mechanics
- 5.2 Elastic material behavior
- 5.3 Plastic material behavior
- 5.4 Viscoelastic material behavior

## **6. Boundary value problems for linear elastic solids**

- 6.1 Field equations for plane strain deformation
- 6.2 Thick walled pressure vessel
- 6.3 Field equations for plane stress deformation
- 6.3 Plate with hole in tension, stress concentration

## **7. Variational methods for elastic solids**

- 7.1 Principle of virtual work
- 7.2 Variational statement of governing equations
- 7.3 Work and energy theorems in solid mechanics
- 7.4 Derivations of field equations for thin plate in bending

## **8. The finite element method for numerical analyses**

- 8.1 Finite elements
- 8.2 Element interpolation functions
- 8.3 Element strains, stresses and strain energy density
- 8.4 Element Stiffness Matrix
- 8.5 Global Stiffness Matrix
- 8.6 Boundary Loading

## **9. Boundary value problems for elastic-plastic materials**

- 9.1 Tension-torsion of thin walled tubes
- 9.2 Plastic limit load
- 9.3 Approximate methods in metal forming

## **10. Failure modes in solid mechanics**

- 10.1 Fracture
- 10.2 Fatigue
- 10.3 Buckling
- 10.4 Large deflections
- 10.5 Plastic collapse

## ***Prerequisites for the course***

### **EN31 Mechanics of Solids and Structures**

Mechanical behavior of materials and analysis of stress and deformation in engineering structures and continuous media. Topics include concepts of stress and strain; the elastic, plastic, and time-dependent response of materials; principles of structural analysis and application to simple bar structures, beam theory, instability and buckling, torsion of shafts; general three-dimensional states of stress; Mohr's circle; stress concentrations. Prerequisites: EN3; AM33.

### ***AM33 Methods of Applied Mathematics***

Mathematical techniques involving differential equations used in the analysis of physical, biological and economic phenomena. Emphasis on the use of established methods, rather than rigorous foundations. I: First and second order differential equations. II: Applications of linear algebra to systems of equations; numerical methods; nonlinear problems and stability; introduction to partial differential equations.

## **Exams**

- **Midterm examination, to be held roughly at mid-semester. More details TBA.**
- **Final examination, to be held as scheduled by the Registrar on 12/21/2013, at 9:00 AM. Location TBA.**

# ***Grading & Collaboration Policies***

## **•Grading Policy**

**Homework:30% (class material: 20%, ABAQUS: 10%)**

**Midterm exam:25%**

**Project: 15%**

**Final: 30%**

• **Collaboration policy:** We encourage discussions on homework and computer assignments: you can learn a lot from working with a group. This means that you are permitted to discuss homework problems and computer assignments with classmates, and are permitted to seek help from other students if you run into difficulties. However, material submitted for grading should represent independent work of its author. It is not acceptable to copy the work of other students, and it is not acceptable for two students to submit identical copies of any part of an assignment.

• **Internet:** You can get help from Internet sources as well. However, you should identify the website(s) from which you got the solution(s)/help.

• **Brown academic honor code**

[http://www.brown.edu/Administration/Dean\\_of\\_the\\_College/academic\\_code/](http://www.brown.edu/Administration/Dean_of_the_College/academic_code/)

## ***Textbook***

**There is no required text for this course.**

- **Free online textbook authored by Prof. Allan Bower:** <http://solidmechanics.org/>

### **Recommended reference books:**

"A first course in Continuum Mechanics" Y.C. Fung. 3rd Edition, Prentice-Hall, 1994.

"Classical and Computational Solid Mechanics" Y.C. Fung & Pin Tong. World Scientific, 2001.

### **Additional references:**

"An Introduction to Continuum Mechanics, 3rd Edition," W. Lai, D. Rubin, and E. Krempf, Butterworth-Heinemann, 1995.

"Advanced Mechanics of Materials," W.B. Bickford, Addison-Wesley, 1998.

"Advanced Strength and Applied Stress Analysis," R.G. Budynas, McGraw-Hill, 1999.

"Advanced Strength of Materials," J.P. Den Hartog, Dover Publishing, 1996

"Introduction to the Mechanics of Continuous Media," L.E. Malvern, (recommended for advanced students only).

"Theory of Elasticity" S.P. Timoshenko and J.N. Goodier (Well written, and contains lots of useful solutions to elastic boundary value problems, but the notation is dated and the book does not cover plasticity or finite element analysis).

"Elasticity," J.R. Barber (A modern and well written introduction to linear elasticity). Kluwer, 2004

## ***Office Hours***

### **Faculty:**

Professor Pradeep Guduru

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### **Teaching Assistant**

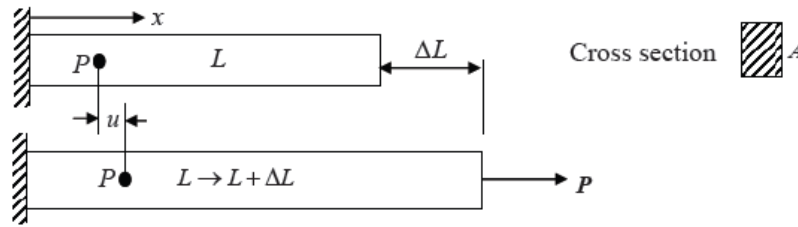
- Max Monn

Email: michael\_monn@brown.edu

Office hours: Wed 5-6pm, Thu 5-6 pm, Location: TBA

## What did we learn in EN 31?

- Axial Loading



Displacement  $u = u(x)$

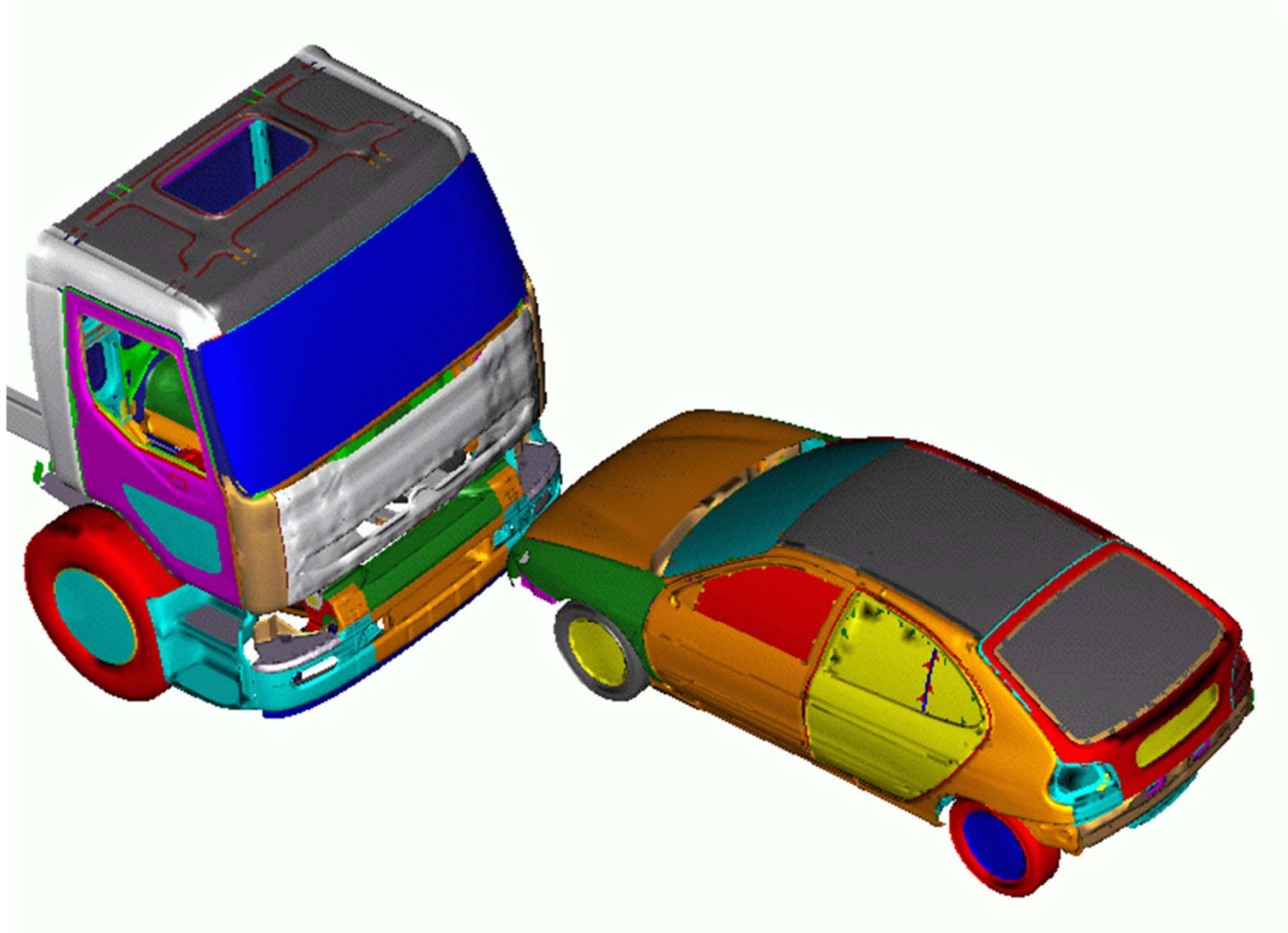
Stress  $\sigma = P/A$  (measure of internal force)

Strain  $\epsilon = \Delta L/L$  (measure of shape change)

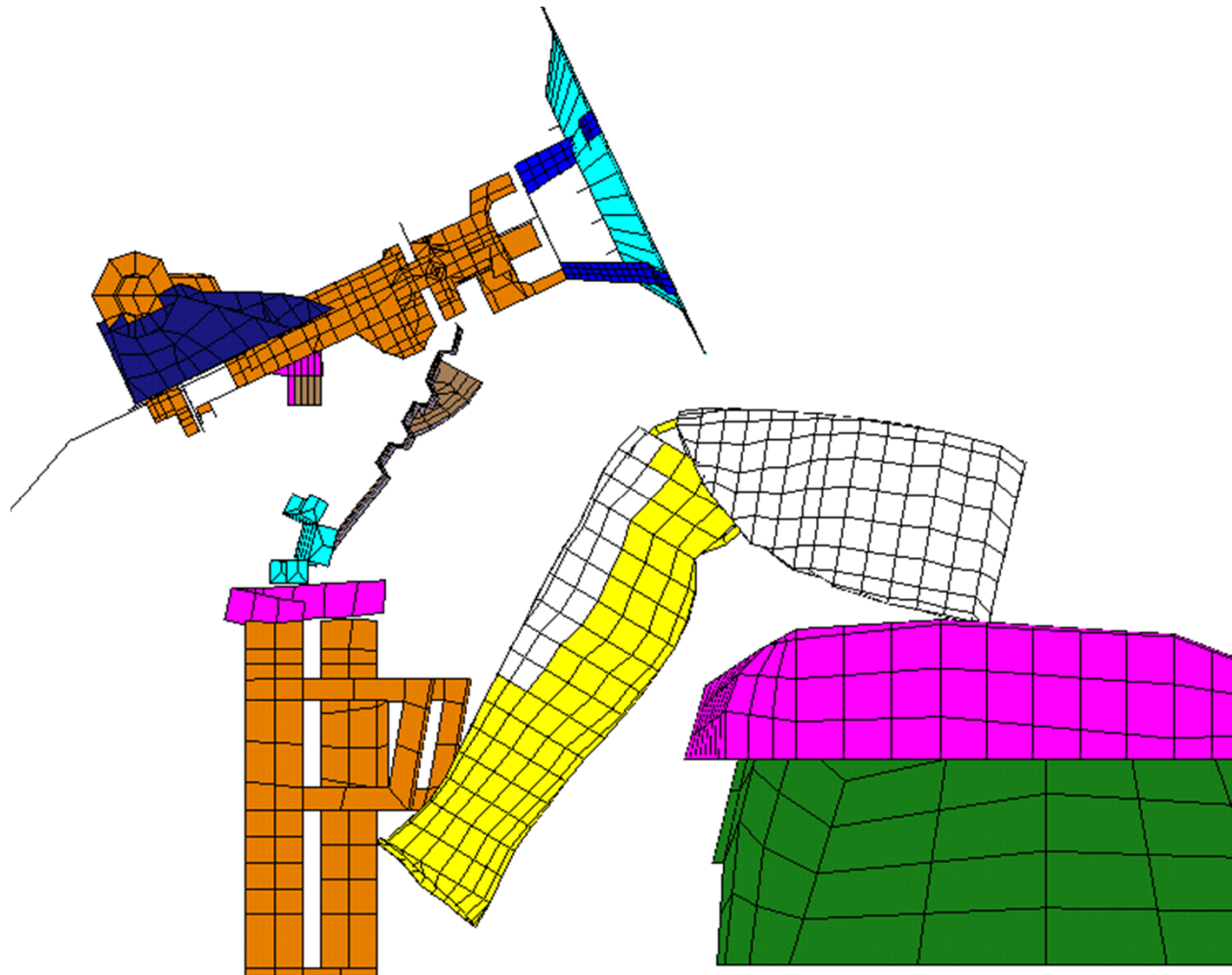
- Torsion of circular shafts (shear stress, shear strain)
  - Bending of beams (stress and deformation)
  - Buckling of columns
  - Failure criteria
- 
- How do we extend the ideas of stress and strain to a solid of complex geometry and complex loading?
  - How do we calculate the stress and strain field in such complex objects so that we can design them to withstand the prescribed loads?



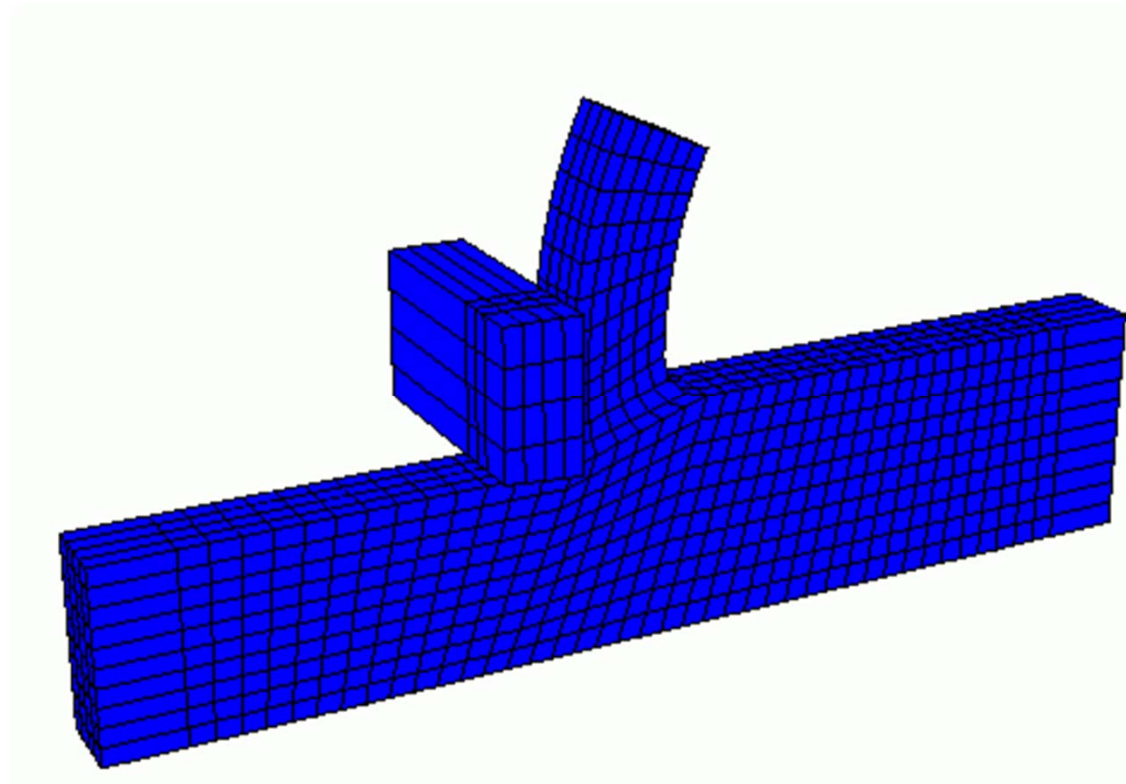
# Application: Crash Simulation



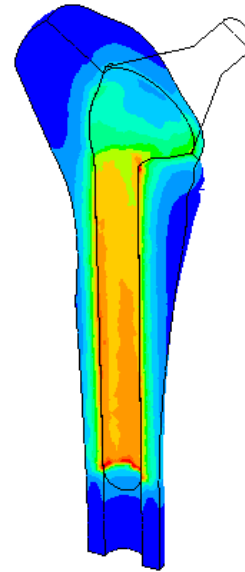
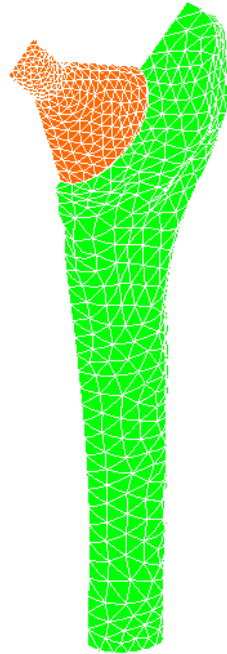
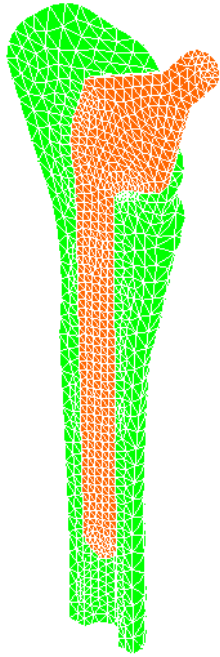
# Knee Protection



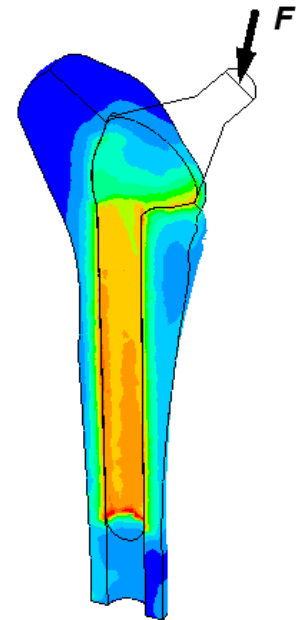
# Milling



# Sample Application: Hip Implant

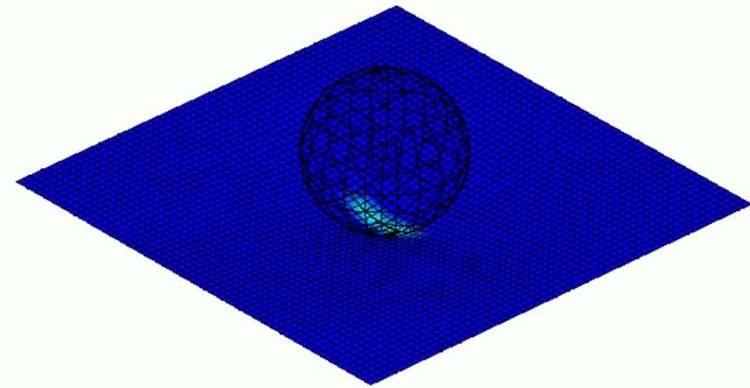
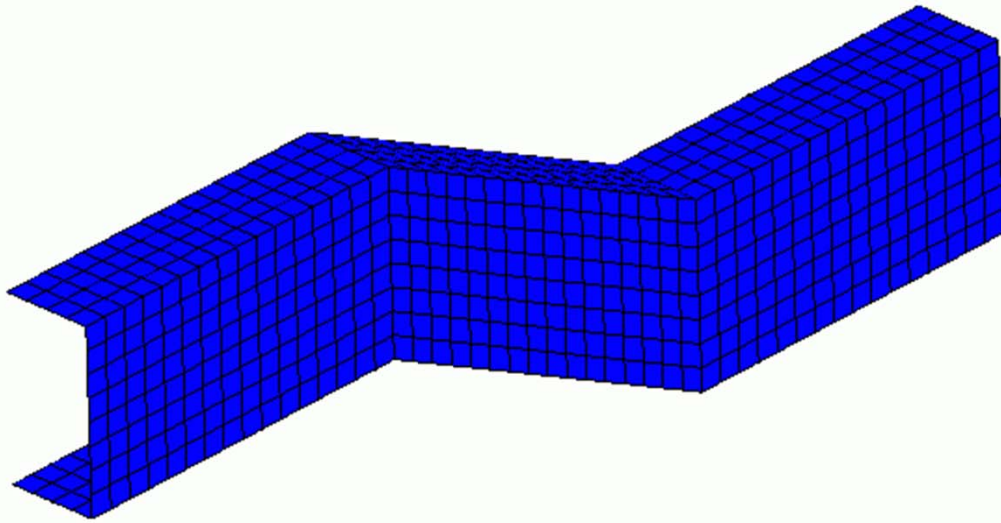


*a) Due to interference fit with implant.*



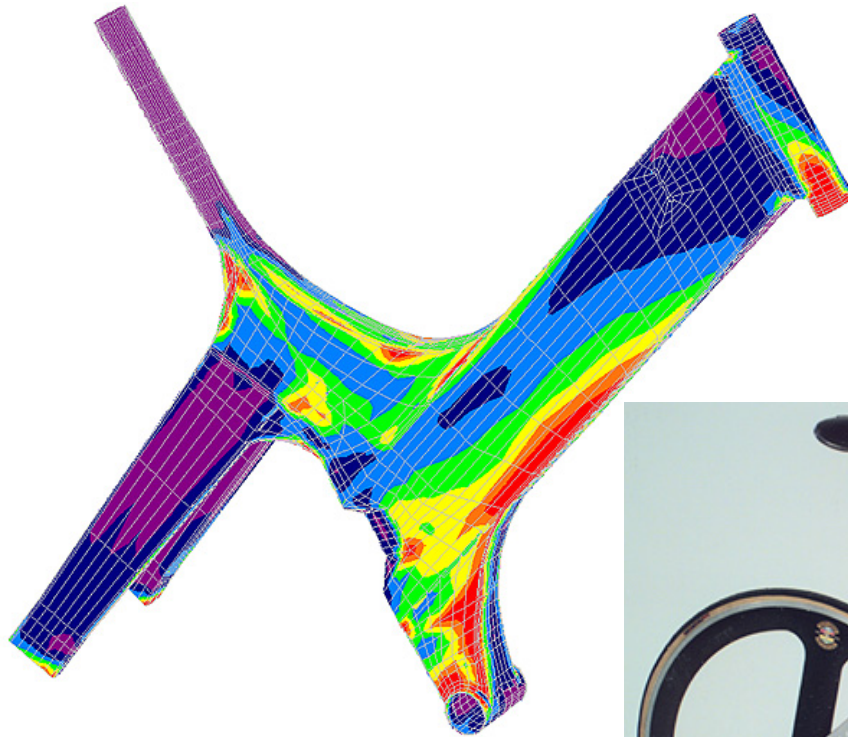
*b) Due to interference fit and applied load.*

# Buckling and Penetration



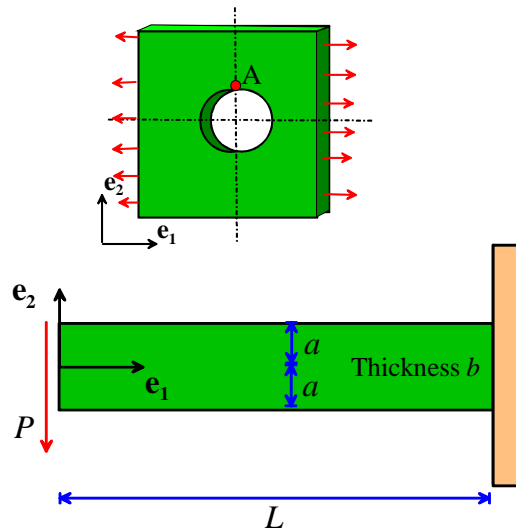


# Bicycle frame design



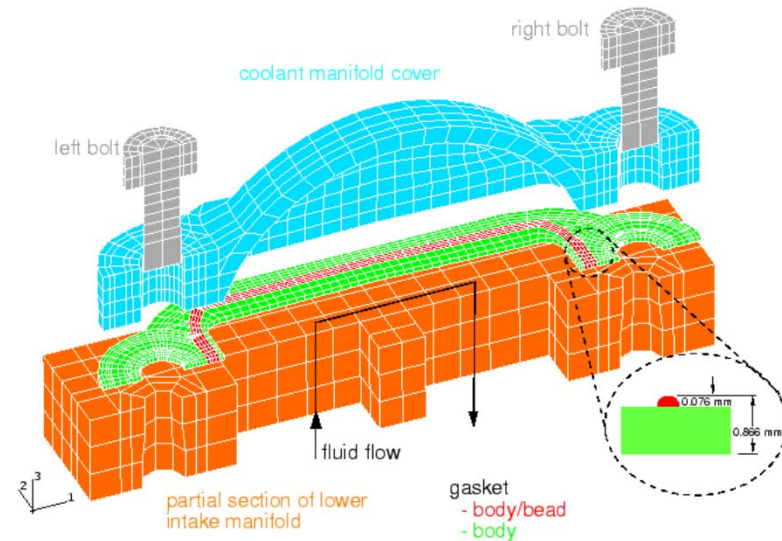
# Tools of the Trade

- Physical Intuition



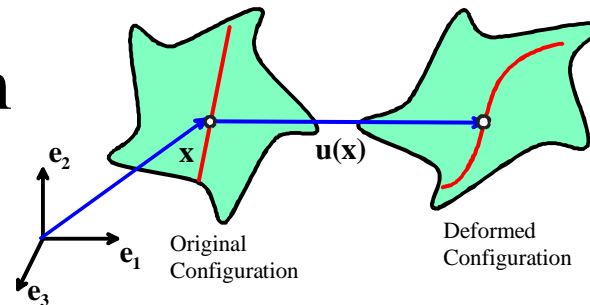
- Solutions to boundary value problems

- Finite Element Analysis

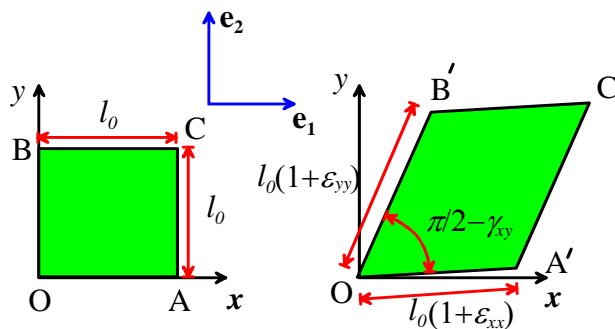


# Concepts in the Mechanics of Solids

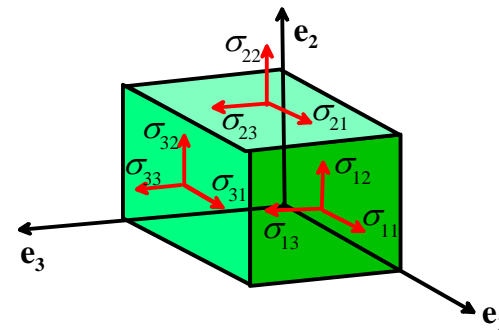
- The Continuum



- Deformation (kinematics)



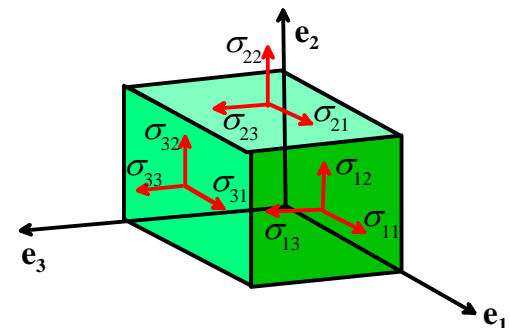
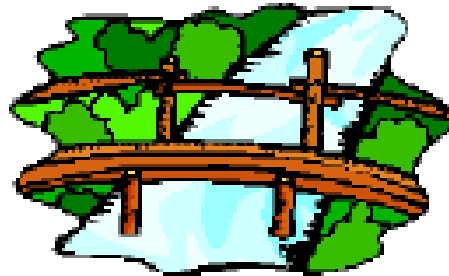
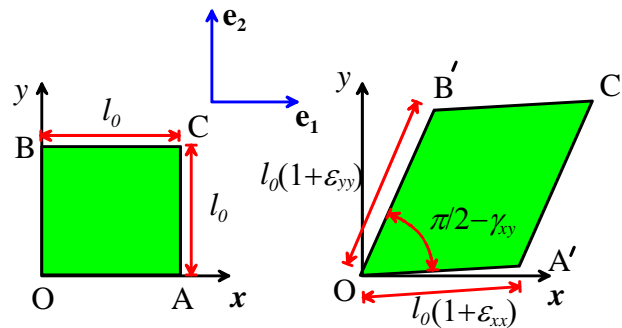
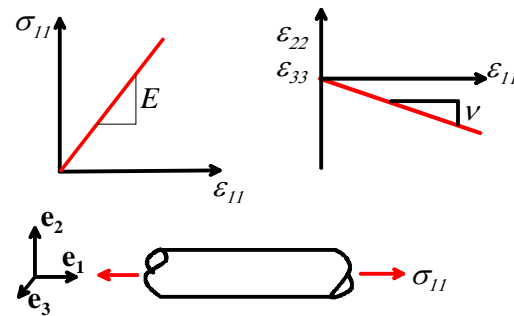
- Stress (kinetics)





# Last But Not Least....

- Material Behavior



- Deformation

- Stress

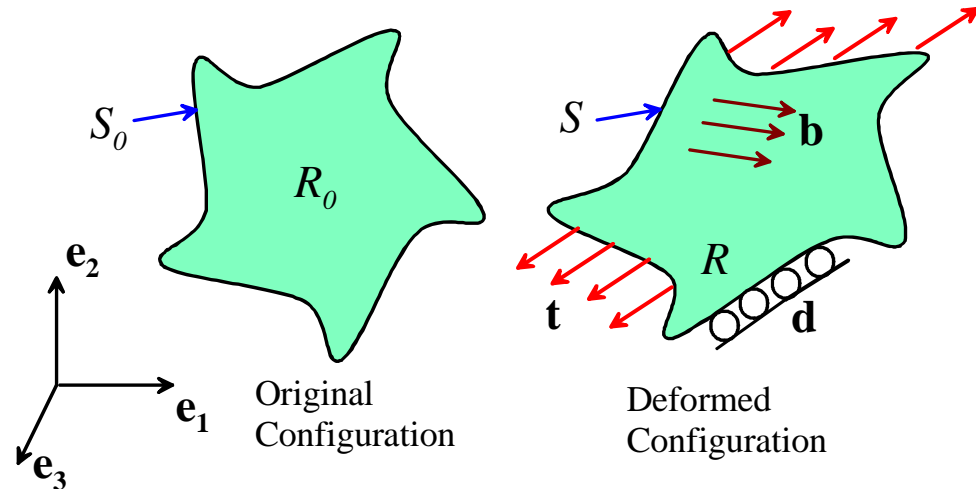
# Boundary Value Problems in Solid Mechanics

## Given

- Geometry
- Material Properties
- Applied Loads
- Applied Temps/Heat
- Initial Conditions

## Compute

- Deformed Shape & stress
- Temperature/Heat Flux



## ***Applications of Solid Mechanics***

**Mechanical engineering**—designing load bearing components for vehicles; power generation and transmission; pressure vessels; engines and turbines

**Civil engineering**—designing foundations or structures (bridges, buildings);

**Aeronautical engineering**—aircraft; space shuttles;

**Geomechanics**—modeling the shape of planets; tectonics; and earthquake prediction;

**Manufacturing engineering**—designing metal and polymer forming processes; machining, etc;

**Biomechanics**—designing implants; bone mechanics; biomimetics; cellular and molecular processes;

**Materials Science**—designing composites; alloy microstructures, thin films, and developing materials processing

**Microelectronics**—designing failure resistant interconnects and packaging;

**Nanotechnology**—stress driven self-assembly on surfaces; manufacturing processes such as nano-imprinting; modeling atomic-force microscope/sample interactions.

**Energy** – Batteries, windmills, light-weight materials, fuel cells, catalysis

# Solid Mechanics in Engineering Design



- Strength?
- Life?
- Deformation?
- Stability?
- Vibrations?



- Material Selection
- Shape Optimization
- Cost
- Manufacturability

